- International System of Units (SI System)
- Base Units
- Derived Units
- Scientific Notation
- Precision
- Accuracy
- Significant Digits ("sig-figs")
- Linear Relationships
- Non-linear relationships
- Inverse Relationships

UNIT I: MATH TOOLS

I. Units and Standards

A. In physics **units** are **standards** for measurement of physical quantities that need clear definitions to be useful

1. **Reproducibility** of experimental results is central to the **scientific method**.

2. To facilitate this we need standards, and to get convenient measures of the standards we need a **system of units**

B. Different systems of units are based on different choices of a set of **fundamental units**

C. The most widely used system of units is the International **System of Units**, or **SI**

1. There are **seven SI base units**

SI BASE UNITS		
Base quantity	**Base Unit**	**Symbol**
Length	meter	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Amount of substance	mole	mol
Electric current	ampere	A
Luminous intensity	candela	cd

2. All other SI units can be **derived** from these base units.

a. **Derived units** are combinations of the base units.

b. Examples:

Speed = m/s,
Energy = joule = kg x m^2/s^2
Force = Newton = kg x m/s^2

3. **SI Prefixes**

a. Uses **decimal** system

b. Used to change SI units by **powers of 10**

c. Prefixes used for all quantities

PREFIXES USED WITH THE SI SYSTEM				
Prefix	**Symbol**	**Multiplier**	**Sci.Notation**	**Example**
femto	f	1/1 000 000 000 000 000	10^{-15}	femtosecond (fs)
pico	p	1/1 000 000 000 000	10^{-12}	picometer (pm)
nano	n	1/1 000 000 000	10^{-9}	nanometer (nm)
micro	µ	1/1 000 000	10^{-6}	microgram (µg)
milli	m	1/1 000	10^{-3}	milligram (mg)
centi	c	1/100	10^{-2}	centimeters (cm)
deci	d	1/10	10^{-1}	deciliter (dl)
kilo	k	1 000	10^{3}	kilometer (km)
mega	M	1 000 000	10^{6}	megagram (Mg)
giga	G	1 000 000 000	10^{9}	gigameter (Gm)
tera	T	1 000 000 000 000	10^{12}	tetrameter (Tm)

II. **Scientific Notation** ($M \times 10^n$ where $1 \leq M < 10$ and n is an integer)

A. Reasons scientists use scientific notation

1. Many measurements are very small or very large. Values take up a lot of space (e.g. distance sun to mars = 227,800,000,000 m written as 2.2278×10^{11}m)

2. Makes easier calculations

B. Converting Units

1. Use **conversion factors**—a relationship between two units (e.g. 1 kg = 1000 mg)

2. It is a multiplier equal to 1

3. This method called the **factor-label method** (allows you to check if you did the math correct)

Example: 465 grams = how many kg?

$$(465\ g) \times (1\ kg/1000\ g) = (465g \times 1\ kg)/1000\ g = 0.465\ kg$$

$$= 4.65 \times 10^{-1}\ kg$$

C. **Addition** and **subtraction** using scientific notation

1. Move decimal point if needed to make exponents the same (e.g. adding 4×10^6 and $4 \times 10^{6)}$

2. Add or subtract values of (**M**) and keep the same (**n**)

($M \times 10^n$ where $1 \le M < 10$ and n is an integer)

Example: 4×10^5 m + 2×10^5 m = 6×10^5 m

D. **Multiplication** using scientific notation

1. Multiply the values of (**M**) (remember - $M \times 10^n$)

2. Add exponents (**n**)

Example: (4×10^5 m) x (2×10^5 m) = 8×10^{10} m

E. **Division** using scientific notation

1. Divide the values of (**M**) (remember - $M \times 10^n$)

2. Subtract exponents (**n**)

Example: (4×10^5 m) ÷ (2×10^5 m) = 2×10^0 m = 2 m

III. **Measurement Uncertainties** (how trustworthy is the data?)

A. **Precision**– the degree of exactness of measurement

1. Depends on the instrument used to measure

2. The precision of a measurement is ½ the smallest division of the instrument)

B. **Accuracy**– describe how well the results of an experiment agree with the standard value.

1. A measurement may be precise but not accurate

2. Accuracy of measurement depends on person making the reading.

3. Accuracy of measuring devices should be checked for regularly and calibrated against accepted standards.

3. **Parallax error**– apparent shift in the position of an object when it is viewed from different angles.

IV. **Significant Digits** *(because precision of measuring devices are limited, number of digits in measurement also limited)*

A. Significant Digit Rules

1. **Nonzero digits are always significant**

2. **All final zeros after the decimal point are**

significant

3. **Zeros between two other significant digits are always significant**

4. **Zeros used solely as placeholders are not significant**

B. Number of Significant Digits indicates precision with which measurement was taken

C. **Addition** and **Subtraction** with significant digits

1. Perform operation

2. Round off result to **least precise** value involved

*Example: 35.002 m + 5.21 m = 40.212 m = **40.21 m***

D. **Multiplication** and **division** of significant digits

1. perform calculation

2. Round product or quotient to number that had least number of significant digits

*Example: 4.500 m x 2.0 m = 9.000 m = **9.0 m***

V. Visualizing Data *("A picture is worth a thousand words")*

A. Graphing Data

1. Analyze data carefully

2. Which value did experimenter change?

a. Called the **independent** or manipulated variable

b. Goes on the **x-axis**

3. The other variable changes with the independent variable

a. Called the **dependent** or responding variable

b. Goes on the **y-axis**

B. **Linear Relationships**

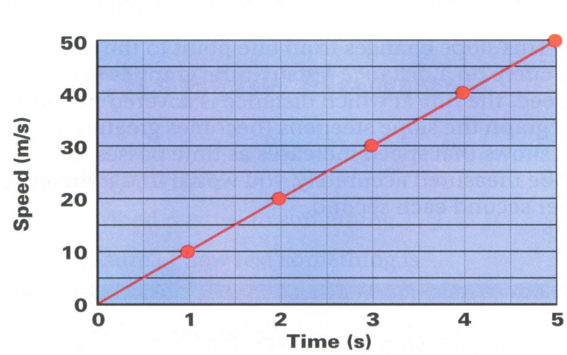
1. two variables are **directly proportional**

2. Forms a **straight line** graph

3. equation: **$y = mx + b$**

a. **m** = **slope** = **rise/run** = **$\Delta y / \Delta x$**

b. b = y-intercept point at which line crosses y axis and where value of x = 0



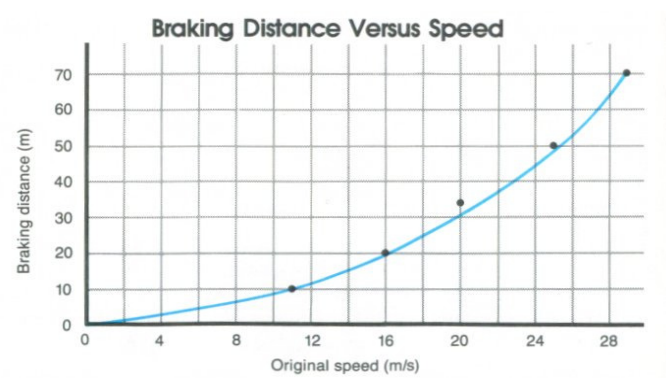
C. **Non-linerar Relationships**

1. **Quadratic relationship**

a. one variable **depends on square of another**

b. Data points **curve upwards**

c. Equation: $\mathbf{y = ax^2 + bx + c}$

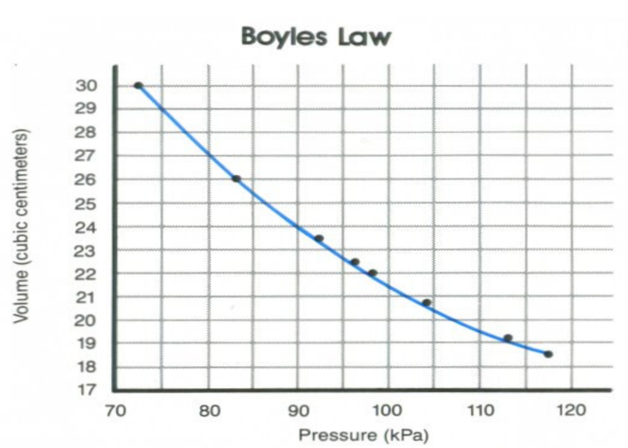


2. **Inverse relationship**

a. One variable **depends on inverse** of the other

b. Data points **curve downwards**

c. Equation: **y = a/x or xy = a**



VI. Rearranging Formulas

A. The subject of a formula

Most engineering students will be familiar with Ohm's law which states that $V = IR$. Here, V is a voltage drop, R is a resistance and I is a current. If the values of R and I are known then the formula $V = IR$ enables us to calculate the value of V. In the form $V = IR$, we say that the **subject** of the formula is V. Usually the subject of a formula is on its own on the left-hand side. You may also be familiar with Ohm's law written in either of the forms

$$I = \frac{V}{R} \quad \text{and} \quad R = \frac{V}{I}$$

In the first case I is the **subject** of the formula whilst in the second case R is the **subject**. If we know values of V and R we can use $I = V/R$ to find I. On the other hand, if we know values of V and I we can use $R = V/I$ to find R. So you see, it is important to be able to write formulas in different ways, so that we can make a particular variable the subject.

B. Rules for rearranging, or transposing, a formula

You can think of a formula as a pair of balanced scales. **The quantity on the left is equal to the quantity on the right**. If we add an amount to one side of the scale pans, say the left one, then to keep balance we must add the same amount to the pan on the right. Similarly if we take away an amount from the left, we must take the same amount away from the pan on the right. The same applies to formulas. If we add an amount to one side, we must add the same to the other to keep the formula valid. If we subtract an amount from one side we must subtract the same amount from the other. Furthermore, if we multiply the left by any amount, we must multiply the right by the same amount. If we divide the left by any amount we must divide the right by the same amount. When you are trying to rearrange, or **transpose**, a formula, keep these operations clearly in mind.

To transpose or rearrange a formula you may

- **add or subtract the same quantity to or from both sides**
- **multiply or divide both sides by the same quantity**

WHATEVER YOU DO TO ONE SIDE…..DO TO THE OTHER SIDE!

Remember ORDER OF OPERATIONS:

Rule 1: Simplify all operations inside parentheses.
Rule 2: Next, do any work with exponents or radicals
Rule 3: Perform all multiplications and divisions, working from left to right.
Rule 4: Perform all additions and subtractions, working from left to right.